

Computer Analysis of the UFO Puzzle

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Introduction

UFO was created by Hiroshi Yamamoto and presented by Nob Yoshigahara in February 1998 at the Third Gathering for Gardner in Atlanta, Georgia. The handout had sample problems using a 9×9 board that turned out to be much larger than necessary for reasonably difficult problems. Nob subsequently reduced the board to 5×5 . Papers with 40 or more sample problems for the 5×5 board have been circulating in the puzzle collecting community since the presentation. Binary Arts will introduce a commercial version called Lunar Lockout in early 2000.

Description of the UFO Puzzle

The standard UFO puzzle uses a 5×5 board, *exiting* pieces and *nonexiting* pieces. Exiting pieces are labeled *X*, *Y* and *Z* and referred to as *letter* pieces. Nonexiting pieces are labeled *1* through *6* and referred to as *number* pieces.

Thousands of problems are possible by placing the pieces on the board in different configurations. The primary objective is to move all of the letter pieces to the center. A secondary objective is to do so in a specified minimum number of *moves*. Letter pieces never begin in the center and must be removed whenever they stop there. Number pieces can begin or stop in the center, but are never removed.

A move, which is the same for letter and number pieces, is made in *steps*. For each step, a piece travels left, right, up or down towards another piece, stopping in the square next to the piece moved towards. Travel is stopped *only* by another piece and must *always* be towards another piece. After stopping, there are 3 possible outcomes:

- (1) The move ends if a letter piece stops in the center. It must be removed from the board immediately and if it is the last, or only, letter piece, the problem has been solved. However, a letter piece may travel *through* the center.
- (2) The move may end and another piece may move.
- (3) The move may continue by turning the piece 90° and traveling towards another piece.

In the sample problem in figure 1, there are 4 possible first moves: Piece *X* can move down to 42 [1 step], down to 42 and right to 43 [2 steps] or down to 42, right to 43 and up to 23 [3 steps]. Piece *4* can move up to 32. Pieces *1*, *2* and *3* cannot move. Note that piece *X* passes through the center in move 1 of the solution, but is not removed until move 3 when it stops there.

11	1	2	14	15
21	X	23	24	25
31	32	×	34	35
41	42	43	3	45
51	4	53	54	55

Start

11	1	2	14	15
21	X	X	24	25
31	32	×	34	35
41	42	43	3	45
51	4	53	54	55

First Move

Solution (3 Moves – 6 Steps)

1. Move *X* down to 42, right to 43 and up to 23.
2. Move *1* down to 42 and right to 43.
3. Move *X* down to the center.

Fig. 1. Sample Problem.

Problem Analysis

Analysis was done for all problems on the 5 × 5 board having 1 letter piece with 1 to 6 number pieces, 2 letter pieces with 1 to 5 number pieces and 3 letter pieces with 1 to 4 number pieces. Experiments with other board shapes and sizes were also done. One of the best was the 7 × 7 English Cross style peg solitaire board. Analysis was done for all problems on this solitaire board having 1 letter piece with 1 to 5 number pieces, 2 letter pieces with 1 to 4 number pieces and 3 letter pieces with 1 to 3 number pieces. Limited analysis was done for more complex problems on both boards.

The analysis programs are written in the PL/I programming language using the professional version of IBM’s PL/I compiler for the Windows operating system. This compiler generates very efficient machine language making it a good choice for compute intensive programs. The analysis was run on an IBM PC compatible with a 333mhz Pentium II processor. The longest running *problem group* (i.e., combinations of letter and number pieces) was group 3–3 for the solitaire board, taking 6 hours.

With problem groups having more letter or number pieces than those completely analyzed, the number of problems increases greatly. Individual problems become so complex (thousands of problems with more than 500 million positions!) that many take several minutes to analyze. For example, it takes several days to solve the problems for a single placement of pieces *X*, *Y* and *Z* in problem group 3–5 for either board. A faster processor would make a difference, but not enough. Furthermore, the problems in these groups tend to be so difficult that most humans would never solve them, and it seems a bit odd to use a computer to create problems that only a computer can solve. Some have been included in the sample problems shown in figures 10 and 11.

The basic analysis is done in two steps. The first step creates all of the problems for a problem group. To reduce the number of problems analyzed, rotations and reflections are

eliminated. Additional measures further reduce the number of problems. All problems where no move is possible are omitted. Next, some simple checks determine if any letter piece is *stranded*. A letter piece is stranded when it is impossible to maneuver it and another piece into squares opposite the center.

The second step attempts to solve all problems for a problem group. The solver program uses a modified depth-first search. It finds all minimum-move, minimum-step solutions.

For many problems, solutions exist where one or more of the number pieces are *insignificant*. That is, they do not move and do not stop other pieces from moving. In figure 2, problem 1a from problem group 1–2 is solved by simply moving piece *X* down. Piece *I*, is insignificant. Excluding rotations and reflections, in this problem there are 12 insignificant placements for piece *I*. If the insignificant piece in problem 1a is ignored, problem 1b from problem group 1–1 is identical.

Problem 2a has 2 solutions with 4 moves and 6 steps. One (**2-D 1-R 4-LU X-RD**) uses all of the number pieces. The other (**3-L 2-D 4-LU X-RD**) does not use piece *I*. Therefore, if the insignificant piece in the second solution for problem 2a is ignored, problem 2b from problem group 1–4 is identical.

For the human solver, insignificant pieces generally make it easier to meet the primary objective of moving all of the letter pieces to the center, while making it harder to meet the secondary objective to do so in a specified minimum number of moves. A good collection of problems will always include some with insignificant pieces to enhance the human solver’s experience. However, because this analysis is dealing with optimum solutions, problems with insignificant pieces are not included in the solution counts in tables 1 and 2.

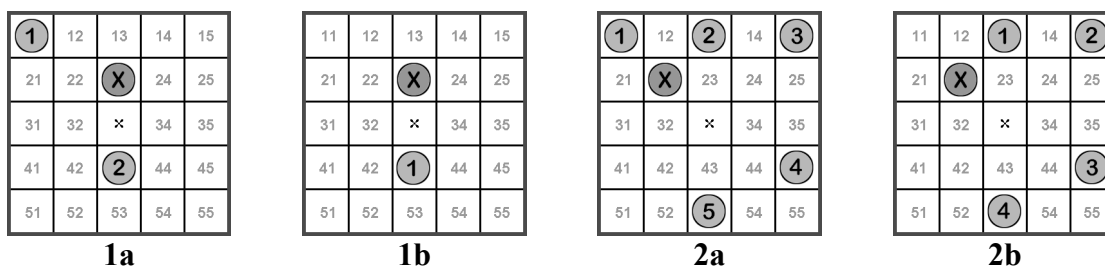


Fig. 2. Problems with insignificant number pieces

For all problems with solutions, the solver program writes text files of the solutions for visual browsing and a compact binary solution file for further analysis. Tables 1 and 2 show a summary of the solver program results.

Group ¹	Problems Possible ²	Problems Analyzed ³	Problems With Solutions ⁴	Max Moves ⁵	Max Steps ⁵
1-1	576	4	2	1	1
1-2	13,248	86	11	2	2
1-3	291,456	1,657	115	5	5
1-4	6,120,576	15,535	1,597	10	12
1-5	122,411,520	85,156	8,045	10	15
1-6	2,325,818,880	321,703	9,185	14	22
2-1	12,696	74	1	2	2
2-2	279,312	2,138	36	5	5
2-3	5,865,552	27,329	1,303	10	13
2-4	117,311,040	192,258	43,101	16	23
2-5	2,228,909,760	889,208	182,107	15	25
3-1	267,168	1,217	0	0	0
3-2	5,610,528	24,190	45	7	8
3-3	112,210,560	232,562	14,836	16	23

Table 1. 5×5 board.

¹Letter and number piece count.

²Including rotations and reflections.

³Excluding rotations, reflections, problems with stranded letter pieces and problems with no moves.

⁴For all problems analyzed excluding solutions with insignificant pieces.

⁵For the best (minimum-move, minimum step) solutions in the problem group.

Group ¹	Problems Possible ²	Problems Analyzed ³	Problems With Solutions ⁴	Max Moves ⁵	Max Steps ⁵
1-1	1,024	9	3	0	0
1-2	31,744	254	20	2	2
1-3	952,320	6,128	274	4	5
1-4	27,617,280	76,171	4,194	11	12
1-5	773,283,840	570,210	43,915	16	20
2-1	30,752	233	3	2	2
2-2	922,560	8,483	97	6	6
2-3	26,754,240	142,847	4,206	10	12
2-4	749,118,720	1,355,196	138,297	17	24
3-1	892,800	5,225	1	3	3
3-2	25,891,200	134,247	253	9	10
3-3	724,953,600	1,718,986	66,784	18	26

Table 2. Solitaire board.

¹Letter and number piece count.

²Including rotations and reflections.

³Excluding rotations, reflections, problems with stranded letter pieces and problems with no moves.

⁴For all problems analyzed excluding solutions with insignificant pieces.

⁵For the best (minimum-move, minimum step) solutions in the problem group.

Solution Analysis

Two additional programs were used to analyze the solutions. One is used to select and sort solutions meeting different criteria. For example, all problems having solutions with a minimum or maximum number of moves or steps can be selected. In addition to moves, steps and position of the pieces, it can use, as selection criteria, the sum of moves and steps, the difference between moves and steps, and moves as a percentage of steps.

The other program is used to analyze number-piece configurations. It processes all of the solutions from the solver program, ignoring letter piece placement. Rotations and reflections of the number-piece configurations are normalized into a single form and looked up a table where an occurrence count is kept. After all solutions have been processed, the table is sorted in ascending or descending sequence to determine the number-piece configurations with the least or most occurrences. The results of the analysis suggest some challenging new problems presented in the following sections.

Configurations with the Most Solvable Problems

When solutions with insignificant number pieces are included, there are several number-piece configurations where piece *X* can be placed in any vacant square except the center and all of the resulting problems are solvable. For the 5 x 5 board, there are 215 from problem group 1–5 and 3,239 from problem group 1–6. For the solitaire board, there is 1 from problem group 1–4 and 6 from problem group 1–5. These are the only analyzed problems groups, for either board, with number-piece configurations where all problems are solvable.

A few of the number-piece configurations with the *most* solvable problems where all number pieces are significant are shown in figures 3, 4 and 5. Remember that letter pieces *never* begin in the center. Answers are at the end of the article.

Configuration 1 in figure 3, from problem group 1–4, has 16 solvable where all number pieces are significant. This is the most of any analyzed problem group for the 5 × 5 board. Can you find the 4 problems where no solution is possible?

Configurations 2 and 3 in figure 3, from problem group 1–5, each have 15 solvable problems where all number pieces are significant. The other 4 problems for each configuration are solvable, but have insignificant number pieces. Can you find them?

Configuration 4 in figure 3, from problem group 1–6, has 13 solvable problems where all number pieces are significant. This is the most for this problem group. The other 6 problems are solvable, but have insignificant number pieces. Can you find them?

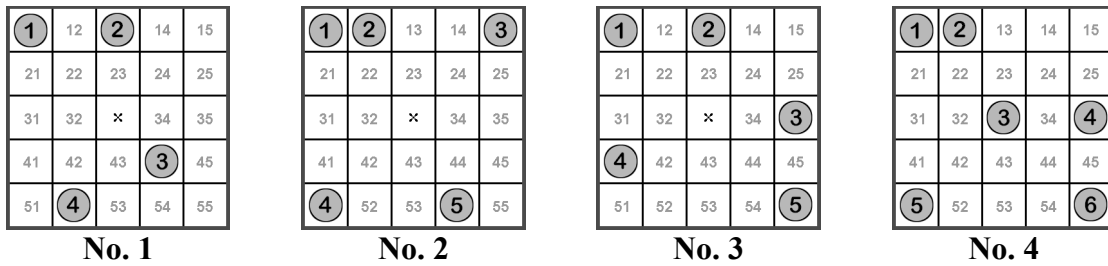


Fig. 3. Place X in any vacant square and solve the resulting problem.

Configurations 1, 2 and 3 in figure 4, from problem group 1–5, all have 23 solvable problems where all number pieces are significant. Each configuration also has 3 unsolvable problems and 2 solvable with insignificant number pieces. Can you find them?

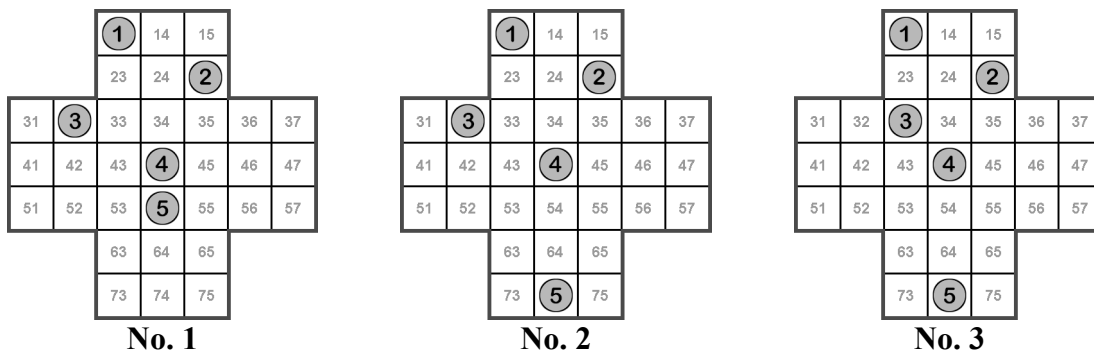


Fig. 4. Place X in any vacant square and solve the resulting problem.

Configuration 1 in figure 5, from problem group 2–5, has 146 solvable problems where all number pieces are significant. The other 25 problems are solvable, but have insignificant number pieces. Can you find them?

Configuration 2 in figure 5, from problem group 2–4, has 155 solvable problems where all number pieces are significant. This is the most of any analyzed problem group for the 5×5 board. Only one of the remaining 35 problems is solvable, but has insignificant number pieces. Can you find it? Can you find the 34 unsolvable problems?

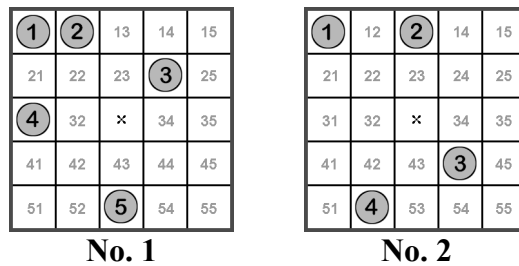


Fig. 5. Place pieces X and Y in any vacant squares and solve the resulting problem.

Configurations with One Solvable Problem

There are numerous number-piece configurations where only one letter piece placement is solvable. Finding the placement is challenging for the simplest problems. For more complex problems, it can be very difficult.

Figure 6 shows 4 of the 59 number-piece configurations from problem group 1–5 where only one placement of piece *X* is solvable and all number pieces are significant. Can you find it?

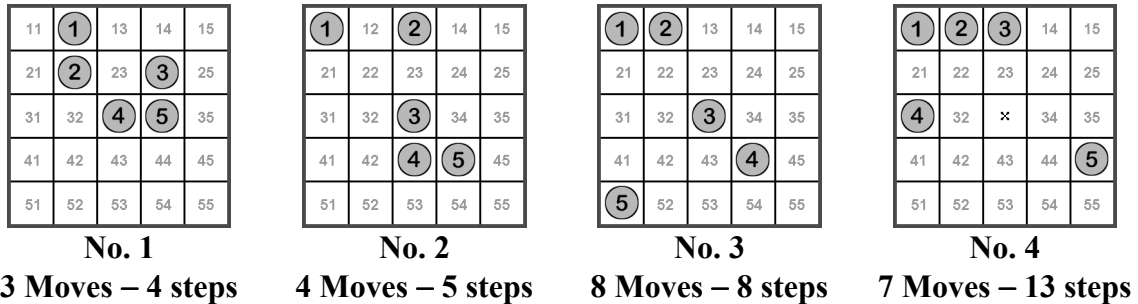


Fig. 6. Find the only placement for piece *X* with a solution.

Figure 7 shows 3 of the 458 number-piece configurations from problem group 1–5 where only one placement of piece *X* is solvable and all number pieces are significant. Can you find it?

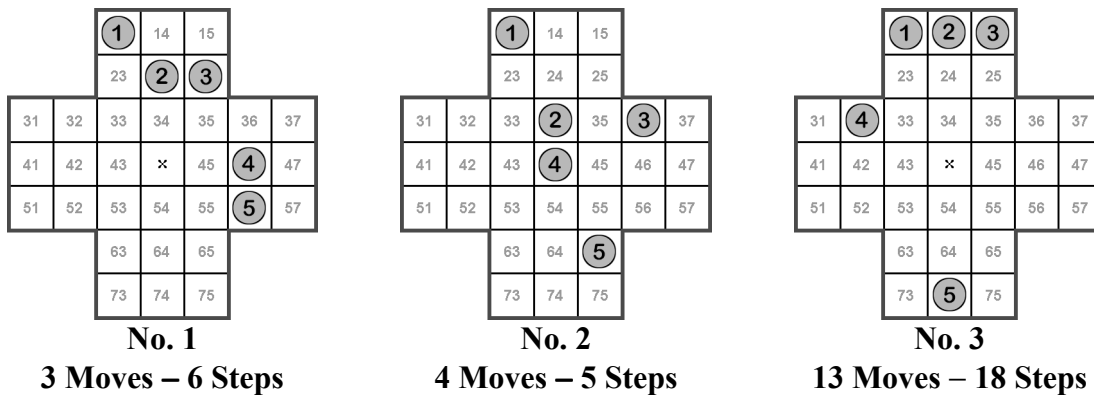


Fig. 7. Find the only placement for piece *X* with a solution.

Figure 8 shows 4 of the 11 number-piece configurations from problem group 2–5 where only one possible placement for pieces *X* and *Y* is solvable and all number pieces are significant. Can you find it?

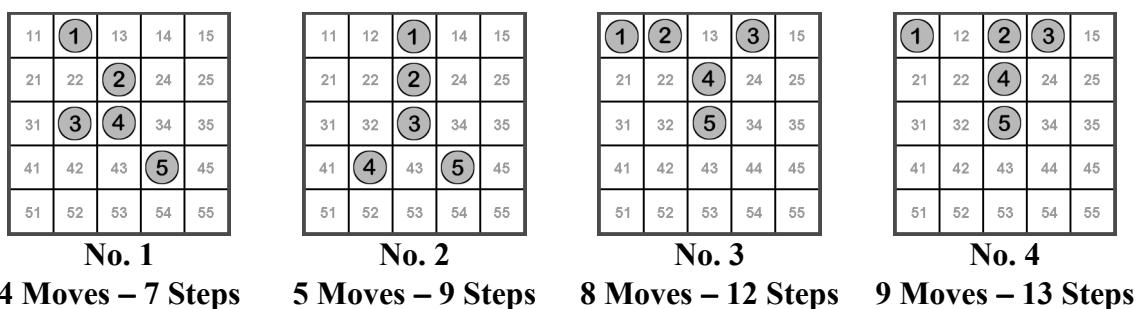


Fig. 8. Find the only placement for pieces *X* and *Y* with a solution.

Figure 9 shows 3 of the 55 number-piece configurations from problem group 2–4 where only one possible placement for pieces *X* and *Y* is solvable and all number pieces are significant. Can you find it?

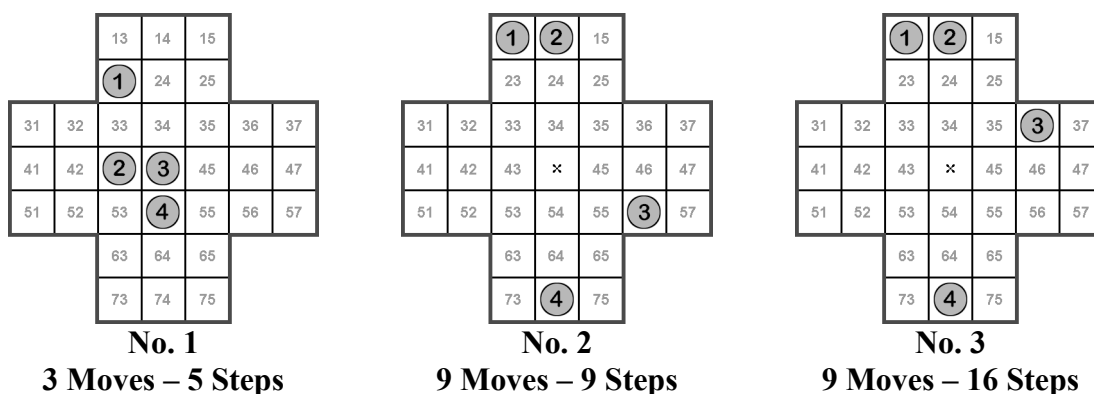


Fig. 9. Find the only placement for pieces *X* and *Y* with a solution.

Conclusions

The UFO puzzle is one of the most interesting new puzzles of its type. While this article has shown that problems exist for the 5×5 and solitaire boards that are, perhaps, too difficult to be fun, the problems with around 5 to 7 moves seem to be fun and challenging for most people.

When released by Binary Arts, Lunar Lockout will have 40 problem cards, 1 exiting piece and 5 nonexiting pieces. Only 3 of the problems require 6 or more moves. Hiroshi Yamamoto selected the 40 problems from computer-produced solutions by Nob Yoshi-gahara. Harry Nelson served as problem tester and critic. Other members of the NOBrain Corps and the Har-e-brain Corps also tested problems. If Lunar Lockout is successful, at least one additional card set, including an additional exiting piece will follow.

Sample Problems

These sample problems vary from simple to extremely difficult. They are not all necessarily representative of those that are fun for most human solvers. Problems 5 to 8 in figure 11 are from problem groups with incomplete analysis.

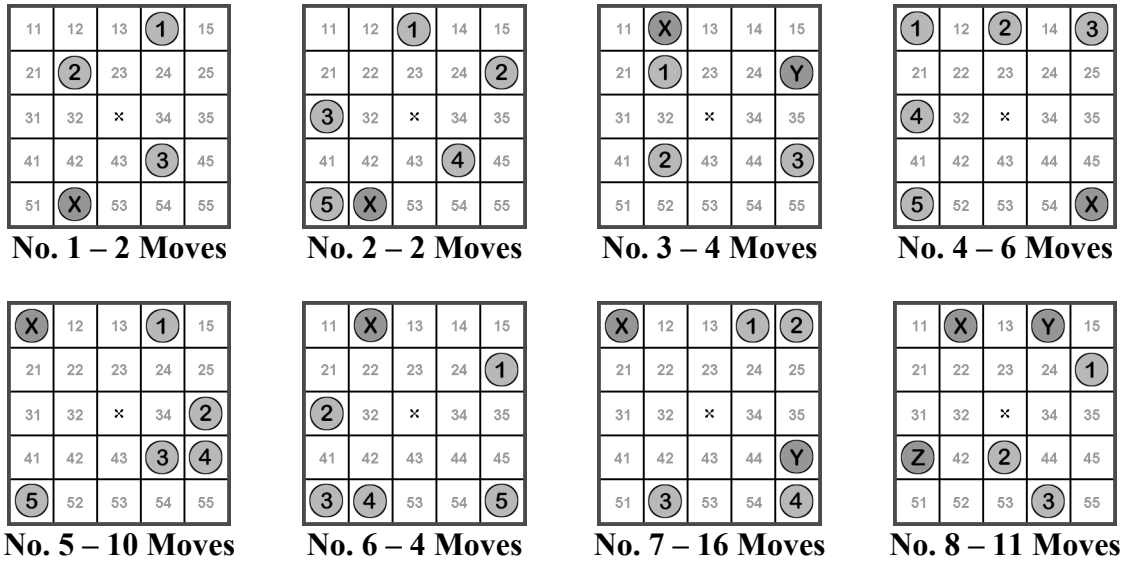


Fig. 10. Sample 5 × 5 problems.

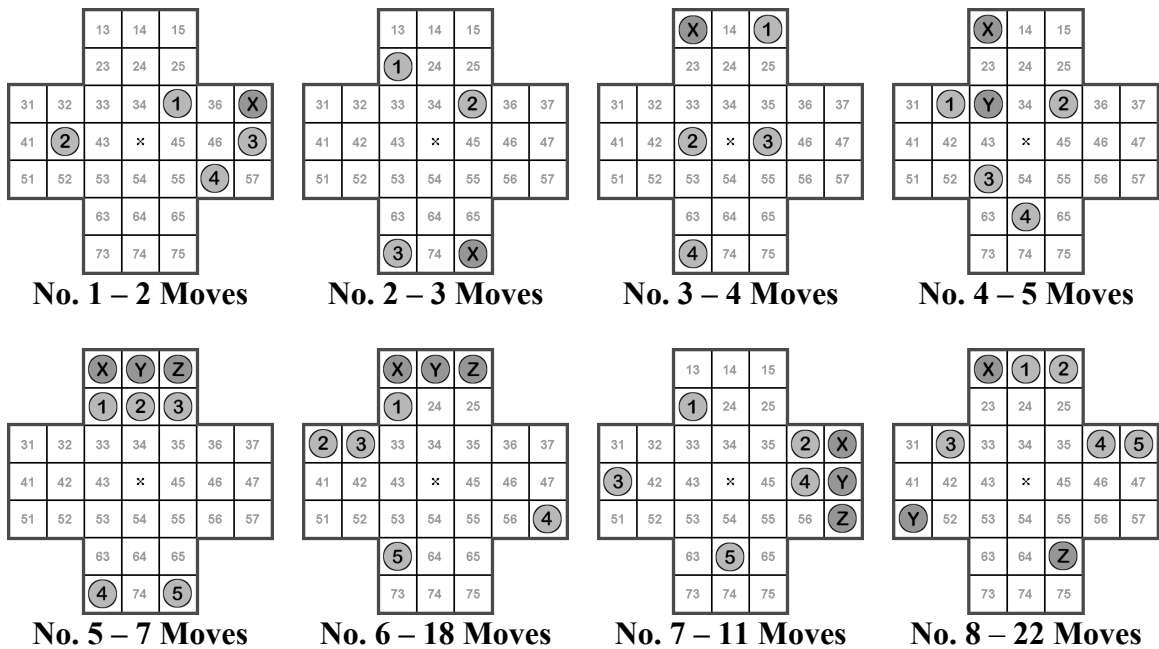


Fig. 11. Sample solitaire problems.

Answers

When a solution is followed by “(1 of n)”, this indicates the solution is 1 of n similar solutions with the same number of moves and steps. Otherwise, it is the only solution with its number of moves and steps.

- Fig. 3
1. With X in 24, 25, 34 or 35, no solution exists
 2. With X in 13, 2 is insignificant – 3-LD 1-DR X-D
With X in 31, 4 is insignificant – 2-RD 1-RD X-R
With X in 32, 4 is insignificant – 2-RD 1-RD X-R
With X in 34, 3 is insignificant – 1-D 5-UL 2-D X-L (1 of 2)
 3. With X in 12, 2 is insignificant – 1-DR 3-DL X-DR (1 of 3)
With X in 22, 2 is insignificant – 1-DR 3-DL X-DR (1 of 3)
With X in 25, 2 is insignificant – 1-D 5-U 3-L X-DL (1 of 2)
With X in 43, 1 is insignificant – 3-DL X-U 4-R X-D (1 of 4)
 4. With X in 22, 2 is insignificant – 1-D 3-R 4-DL X-DR (1 of 5)
With X in 23, 2 is insignificant – 1-D 3-R 5-RU 1-R X-D (1 of 3)
 5. With X in 41, 1 is insignificant – 3-R 4-DL 5-RUL 4-U X-RUR (1 of 3)
With X in 53, 6 is insignificant – 1-D 4-L 3-DLU X-LUR (1 of 8)
With X in 31, 1, 2, 5 and 6 are insignificant – 3-R X-R
With X in 32, 1, 2, 5 and 6 are insignificant – 3-R X-R
- Fig. 4
1. With X in 31, 64 or 74, no solution exists
With X in 23, 1 is insignificant – X-R 4-UL X-D
With X in 24, pieces 1 and 2 are insignificant – 4-UL X-D
 2. With X in 31, 54 or 64, no solution exists
With X in 23, 1 is insignificant – X-R 5-U 4-UL X-D (1 of 2)
With X in 24, pieces 1 and 2 are insignificant – 5-U 4-UL X-D
 3. With X in 23, 24 or 34, no solution exists
With X in 54, 5 is insignificant – 1-DR 4-U X-U (1 of 2)
With X in 64, 5 is insignificant – 1-DR 4-U X-U (1 of 2)
- Fig. 5
1. With X and Y in 13-21, 13-22, 13-23, 13-41, 13-43, 13-51, 14-43, 15-43, 21-22, 21-23, 21-43, 22-23, 22-43, 23-25, 23-43, 32-43, 34-43, 35-43, 41-43, 41-51, 42-43, 43-44, 43-45, 43-52 and 43-54, one piece is insignificant.
 2. With X and Y in 32-34, 3 is insignificant – 1-RD X-R 4-U Y-L (1 of 4)
With X and Y in 21-34, 21-35, 21-45, 23-24, 23-25, 23-34, 23-35, 24-25, 24-34, 24-35, 24-43, 24-45, 24-53, 24-55, 25-34, 25-35, 25-43, 25-45, 25-53, 31-45, 34-35, 34-41, 34-43, 34-45, 34-51, 34-53, 34-55, 35-41, 35-43, 35-45, 35-51, 35-53, 35-54 and 45-51 no solution exists.
- Fig. 6
1. X in 31 : 2-R 4-LU X-R
 2. X in 41 : 1-D 3-L 4-U X-RU
 3. X in 14 : 2-R X-D 3-U 2-L 1-D 4-L 2-D X-L (1 of 4)
 4. X in 52 : 2-DR 1-RD 2-L 5-L 2-U 1-RUL X-URU (1 of 3)

- Fig. 7
1. X in 53 : 5-L 2-DR X-URD
 2. X in 42 : 2-R 4-LU 5-U X-R (1 of 6)
 3. X in 35 : 2-D 3-D X-LUR 2-ULU X-D 2-R X-U 4-RU X-D 2-D 4-R 2-U X-U (1 of 16)
- Fig. 8
1. X in 41, Y in 53 : 3-U 4-DLU X-RU Y-U (1 of 2)
 2. X in 21, Y in 25 : 2-LD X-RD 3-U X-L Y-LDL (1 of 4)
 3. X in 51, Y in 54 : 1-D X-R 5-DLU 2-R 4-D Y-ULD 4-U X-U (1 of 4)
 4. X in 51, Y in 54 : 2-L 1-D X-R 5-DLU 2-R 4-D Y-ULD 4-U X-U (1 of 4)
- Fig. 9
1. X in 25, Y in 46 : 3-RU X-LD Y-L (1 of 2)
 2. X in 36, Y in 43 : 1-D 2-D 3-U X-L 2-U 4-U 2-R X-D Y-R (1 of 12)
 3. X in 53, Y in 55 : 1-D X-RU 4-URDL X-D 2-D X-RD 4-UR X-L Y-UL
- Fig. 10
1. 1-D X-UR
 2. 5-URURDL X-URURDL
 3. 1-D 2-R X-DRDL Y-DL
 4. 2-R 3-D X-L 5-URU 2-LDRUL X-URUL (1 of 2)
 5. 1-L X-D 3-L 4-L 3-U X-R 3-D 1-D 3-R X-UR
 6. 4-R 5-ULURD 4-LURDL X-DRDL
 7. Y-U 1-L 2-L 1-D X-R 1-UR X-D 3-R 1-DLU 4-U Y-LDL 1-D 2-D 1-RD Y-U X-RU (1 of 8)
 8. Y-D 2-LURD Y-LUR 2-DLUR Y-D 1-LD Y-L X-DR 1-D Y-RU Z-RU (1 of 3)
- Fig. 11
1. 3-L X-LDL
 2. X-L 3-UR X-U
 3. 1-L 4-U 2-RU X-DR (1 of 2)
 4. 3-U Y-RD X-DRD 1-R Y-U
 5. 4-R 2-D 5-U 3-LD Y-D X-RD Z-DLD (1 of 4)
 6. 1-DR X-D 1-L 4-L 1-U X-RU 3-RDRU X-L 1-D Y-D X-RD 1-L 3-ULD Z-DL X-RU 3-DR X-D Y-D (1 of 14)
 7. 4-L Y-LUR X-DLURDR 2-DLURD X-LULURD Z-LULUR 2-U 4-RU 2-D Z-D Y-LD (1 OF 2)
 8. 2-DL 4-L X-D 4-R 3-RDLU 4-L 1-D 4-R 3-RD 2-RU 4-L 5-L 3-L 4-DLU 2-RDL 4-D X-DRD 2-RUR 3-RU Y-RU X-UL Z-UL (1 OF 15)

Acknowledgments

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